



INTERNATIONAL VALUATION STANDARDS COUNCIL

THE VALUATION OF EQUITY DERIVATIVES

EXPOSURE DRAFT

Comments on this Exposure Draft are invited before 30 November 2014. All replies may be put on public record unless confidentiality is requested by the respondent. Comments may be sent as email attachments to:

CommentLetters@ivsc.org

or by post to IVSC, 1 King Street, London EC2V 8AU, United Kingdom.

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International Valuation Standards Council
1 King Street
London EC2V 8AU
United Kingdom
Email: ivsc@ivsc.org
www.ivsc.org

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Introduction to Exposure Draft

This is the second exposure draft of proposed guidance on the valuation of equity derivatives issued by the IVSC Standards Board (the Board). Having considered the feedback received on the initial exposure draft, the Board is proposing a number of changes to the proposed guidance.

A number of respondents considered the original exposure draft to be too academic in its approach with a lack of guidance on the practical implications of the theories discussed. In this revised draft there is more emphasis on practical considerations. In line with this, the also Board determined that the inclusion of formulae was not appropriate. Apart from the challenges caused by different academic sources notating the same formulae differently, the guidance is not intended to be an educational resource. All formulae have been removed in favour of a qualitative explanation of the underlying principles.

In response to comments received, this revised Exposure Draft includes a discussion on derivative strategies, although since the valuation of a strategy requires the valuation of its component parts the focus remains on the valuation of the individual instruments. Another common request was for more guidance on the applicability of the models and resolution methods discussed to different types of option. This is potentially a controversial area as models are constantly evolving and it is not the role of the standard setter to stipulate whether a specific model or resolution method is suitable for use in a given situation or not. With this in mind, the latest draft includes a non-prescriptive table that indicates, at a high level, which types of model and resolution method may be applied to different types of option.

This proposed Technical Information Paper is the first of a planned series of similar papers to provide guidance on the valuation of derivatives based on different asset classes, ie fixed income, credit, foreign exchange, commodities, asset backed securities and hybrid products. Because this paper is the first in the series the Board considered that public comment should be invited on this second draft before the paper is considered for final issue.

Questions for Respondents

The IVSC Standards Board invites responses to the following questions. Not all questions need to be answered but to assist analysis of responses received please use the question numbers in this paper to indicate to which question your comments relate. Further comments on any aspect of the Exposure Draft are also welcome.

Notes for respondents:

In order for us to analyse and give due weight to your comments, please observe the following:

1. Responses should be made in letter format, where appropriate on the organisation's letter heading. Respondents should indicate the nature of their business and the main purpose for which they either value, or rely upon the value of, equity derivatives.
2. Comments should not be submitted on an edited version of the Exposure Draft.
3. Unless anonymity is requested, all comments received may be displayed on the IVSC website.
4. Comments letters should be sent as an email attachment in either MS Word or an **unlocked** PDF format and no larger than 1MB. All documents will be converted to secured PDF files before being placed on the web site.
5. The e mail should be sent to commentletters@ivsc.org with the words "Equity Derivatives" included in the subject line.
6. Please be sure to submit comments before the 30 November 2014.

Questions

The majority of respondents to the previous Exposure Draft suggested that strategies be included in the guidance. This draft includes a section discussing strategies in paras 24 – 25. As strategies are a combination of individual products of value the Board has not proposed guidance on how strategies may be valued

1. Do you agree with this approach? If not please explain what alternative you would prefer.

Some respondents to the previous Exposure Draft indicated that they found formulae improved the comprehension of valuation models when combined with a qualitative explanation. However, the Board considers that there are a number of practical problems involved in including formulae. These include accounting for relationships that cannot be expressed through a concise mathematical expression, what style of notation to use, and whether the inclusion of formulae would be consistent with the intention that this is practical guidance, not an academic text or training manual. For the above reasons, the Board has provisionally agreed to exclude formulae from the guidance and to rely on a descriptive narrative of the concepts.

2. Do you agree with this approach? If not, how do you believe formulae should be incorporated into the draft?

A majority of respondents to the previous Exposure Draft indicated that they would like more guidance on the applicability of models to different of products. The Board has since added paras 82 – 83, which provide a high-level overview of which valuation models can be applied to different option products, as well as some commonly used methods of model resolution.

3. Do you believe the addition of this section helpful? If not, please specify your reasons.

The Model section of the paper is currently grouped into three types of model: Black Scholes and its extensions, Alternative-Diffusion models and Jump-Diffusion models.

- 4. Do you agree with this categorisation? If not, please indicate the alternative you would prefer.**

The “model calibration and input selection” section from para 72 – 77 offers brief guidance for some of the most common inputs encountered in a valuation.

- 5. Are there any omissions which you believe should be included in the proposed guidance?**
- 6. Are the inputs discussed at an appropriate level of detail? If not, please provide suggestions for how these could be improved.**

Paras 78 – 81 discuss “model implementation”.

- 7. Do you find the discussion on model implementation helpful? If not, please explain how you believe this could be improved.**

Please consider the paper as whole.

- 8. Are there any additional aspects of the valuation of equity derivatives that you believe could usefully be added? If so, please explain what these are.**

Exposure Draft

The Valuation of Equity Derivatives

Technical Information Papers

Technical Information Papers (TIPs) support the application of the requirements in other standards. A TIP will do one or more of the following:

- provide information on the characteristics of different types of asset that are relevant to value,
- provide information on appropriate valuation methods and their application,
- provide additional detail on matters identified in another standard,
- provide information to support the judgement required in reaching a valuation conclusion in different situations.

A TIP may provide guidance on approaches that may be suitable but will not prescribe or mandate the use of a particular approach in any specific situation. The intent is to provide information to assist an experienced valuer decide which is the most appropriate course of action to take.

A TIP is not intended to provide training or instruction for readers unfamiliar with the subject and will be primarily focussed on practical applications. A TIP is not a text book or an academic discussion on its subject, and neither will it endorse or reference such texts.

Contents

Introduction and Scope	7
Definitions	7
The Equity Derivatives Market	8
Equity Derivative Products	9
Forwards and futures.....	9
Equity Swaps.....	9
Options	9
Structured Products.....	12
Corporate issues.....	12
Volatility derivatives.....	13
Strategies	13
Valuation of forwards and futures	14
Valuation of swaps	14
Valuation of options	15
Black Scholes model.....	15
<i>Black Scholes Model Resolution</i>	16
<i>Limitations of Black Scholes</i>	17
Alternative-diffusion models	17
<i>Time dependent volatility models</i>	17
<i>Local volatility models</i>	18
<i>Stochastic volatility models</i>	18
Jump-diffusion models	19
The Greeks – Sensitivity Measures for Derivatives	19
Model resolution	20
Analytical solution	20
Monte Carlo	20
Trees.....	21
Finite difference.....	21
Numerical integration.....	21
Model calibration and input selection	22
Interest Rate	22
Volatility	22
Correlation.....	22
Dividends.....	22
Holding Costs	23
Implementation	23
Option valuation models and their applicable products	23
Black Scholes	23
Alternative-Diffusion.....	24
Jump-Diffusion.....	24
Conclusion	25

Introduction and Scope

1. This TIP forms part of the International Valuation Standards (IVSs) and provides information and guidance to support the consistent application of the principles and requirements in the standards.
2. This guidance should be read in conjunction with the other sections of the IVSs, in particular the *IVS Framework*, *IVS 101 Scope of Work*, *IVS 102 Implementation*, *IVS 103 Reporting* and *IVS 250 Financial Instruments*. The guidance provided in this TIP is intended to increase the confidence and trust of those who rely on valuation by increasing transparency and consistency. It does this by describing the nature of equity derivatives and the underlying principles on which various valuation methods for these products are based.
3. The guidance in this TIP is intended to be broadly applicable to valuations on which third parties will rely, eg those prepared for financial reporting, fund management or calculating regulatory capital. However, the guidance is subject to any specific provisions that may apply in any applicable accounting standard or regulation governing the purpose for which the valuation is prepared.
4. Equity derivatives are derivative contracts where the underlying asset is equity¹ or an equity based product that is publically traded. Whilst many common equity derivatives are discussed within this paper, it is not intended to be a comprehensive library of such products. Other products may exist and although valuation approaches discussed in this TIP may be applicable, the IVSC makes no assertion as to the relevance of this guidance to such products.
5. The paper discusses the equity derivative market and its associated products. Guidance is then provided on generally accepted valuation models and the products to which they can be applied, as well as model resolution, calibration, implementation and input selection.
6. The guidance in this TIP identifies approaches and actions that may be appropriate in typical situations but does not preclude the possibility that alternatives may be equally or more appropriate depending upon the nature of the product or the purpose for which the valuation is required. Selecting the most appropriate approach and implementing it correctly requires professional judgement, objectivity and competence, see *IVS Framework* paras 1-5.

Definitions

7. The following definitions apply in the context of this TIP. Similar words and terms may have alternative meanings in a different context. The IVSC's International Glossary of Valuation Terms provides a comprehensive list of defined words and terms commonly used in valuation, together with any alternative meanings.

Cash flow	An exchange of monies between counterparties.
Coupon	Interest payment made throughout the life of an investment, similar to a dividend.
Deterministic	A process in which no randomness or variables are involved.

¹ In the context of this paper "equity" is used synonymously with "stock" and "share"

Dividend	A regular payment from a company to its equity holders.
Implied volatility	A measure of a financial instrument's variability in value, which is equal to the value inferred from a pricing model that returns a value equivalent to the current market price.
Leg	A side of a swap agreement.
LIBOR	London inter-bank offered rate.
Maturity	The last day that an options or futures contract is valid.
Notional	The face value of an instrument used to calculate payments between the counterparties.
Path-Dependent	An option where the strike price is based on fluctuations in the value or price of an underlying asset during the contract term.
Payoff	The final payment or other financial gain that accrues to the beneficiary.
Stochastic	A process involving a random variable or series of variables.
Strike Price	The price at which an option can be exercised.

The Equity Derivatives Market

8. Equity derivatives are contracts based on the value of or changes in the value of an underlying asset that can be a single equity, a basket of equities, an index based on a basket of equities or the return of a strategy on one or more of those instruments. Equity derivatives can be settled in cash, physically, or a combination of the two. Physical settlement involves the transfer of the relevant underlying equities between parties and fractional equity amounts tend to be settled in cash. Derivative contracts will specify the type of settlement required as well as the circumstances under which it will occur.
9. Equity derivatives are either exchange traded or dealt in the over the counter (OTC) markets.
 - a) Exchange traded contracts tend to be the simplest products and are standardised with respect to contract lengths, amount of underlying equities, price and settlement procedures. This reduces the number of unique contracts and maximises liquidity. Exchange traded contracts also reduce counterparty credit risk as parties to the contract are required to post collateral and their positions are regularly marked to market by the exchange.
 - b) OTC contracts are unique arrangements between parties. Whilst also existing in simple forms, they are often more complicated than those traded on exchanges. As there is no intermediary exchange each counterparty is subject to the risk of the other defaulting. In practice OTC contracts are almost always transacted under a netting agreement² or with a central counterparty (CCP) whereby all contracts with the counterparty are offset in the event of default resulting in a singular claim by one of the

² The ISDA master agreement, developed by the International Swaps and Derivatives Association is the most commonly used master contract for derivative transactions.

parties. The credit support annex (CSA) to a master netting agreement further reduces credit risk by setting terms for posting collateral relating to positions traded under the agreement.

10. When the value of a derivative is adjusted to consider the credit risk of the counterparty it is referred to as a credit valuation adjustment (CVA) and as a debit valuation adjustment (DVA) when considering the entities' own credit risk. For more information readers are referred to IVS 250 *Financial Instruments* and the current IVSC exposure draft on "*Credit and Debit Valuation Adjustments*".

Equity Derivative Products

11. The main types of derivative product are described below.

Forwards and futures

12. A forward contract is an agreement to purchase or sell an underlying equity at a future date at today's predetermined price; these are OTC contracts. Futures are their exchange-traded equivalents with standardised maturities and lot size, where daily fluctuations in contract price must be settled by the relevant counterparties (marked-to-margin).
13. Forwards and futures are also called "delta-one products" because the change in their price is equal to the change of underlying asset's price. For market risk hedging purposes, this implies that they may be hedged with one unit of the underlying asset.

Equity Swaps

14. A swap is a contract where parties agree to exchange payments at predetermined future dates. In an equity swap there are typically two legs, one, often called the financing leg, is based on an interest rate such as LIBOR and the other is based on the price of an equity or equity index. Whilst this is the most common arrangement an equity swap can also have two equity based legs.
15. There are many varieties of equity swap, however for the most part differences between contracts are slight and they can be grouped into overarching categories. The two most common categories are:

Quanto swap	The currency of the underlying equity is different to the currency of the swap.
Total return swap	One party pays the equivalent of the gain (or receives the loss) arising from a change in the value of an underlying plus any dividends, the other party pays a fixed or variable rate.

Options

16. An option contract provides the holder with the right, but not the obligation, to buy or sell an equity at a specified strike price, on or before a specified date. The cost of an option is referred to as the premium. The purchase of a call option grants the holder the right to buy an equity and a put option the right to sell an equity. The simplest options can have a single equity as the underlying; however they can also be linked to multiple equities or indices. Due to this range of potential underlying, options appeal to both speculators and those looking to hedge exposure to future price changes.

17. The option holder will choose to exercise a call option when the market price of an equity is above the strike price and exercise a put option when the market price is below the strike price. An example is where an investor pays \$1 for a call option to buy one equity in a company with a strike price of \$20 in three months' time. If the market price is \$25 in three months' time the investor will exercise the option and the return will be \$4 ($\$25 - \$20 - \1). If the market price were below \$20 the investor would choose not to exercise and will have lost the premium of \$1. If the price is greater than \$20 and less than \$21, for example \$20.50, exercising the option will reduce the loss to \$0.50.

18. There are a wide variety of different option types and some of the more common types are described below.

American	The holder may exercise the option at any point up to and including maturity. These are generally more valuable than European options as the holder has more rights but the holder also has to determine when it is most advantageous to exercise their right.
Asian (Average)	Instead of depending on the underlying asset price at a single time, the payoff depends on the average price over a period of time. The averaging period may be over the entire life of the contract or just a portion. These are generally less valuable than other options as they average out the effects of volatility which is the primary driver for option values. Asian (average price) options are typically not physically settled, since the equity price at the settlement date is unlikely to equal the average price calculated per the option terms.
Barrier	This type of option is known as a barrier as the potential payoff is contingent on whether or not the price of the underlying equity meets or crosses a specified target level (or levels in the case of a double barrier option). This underlying price level is often referred to as a knock-in or knock-out barrier that may be observed at particular points or throughout the life of the trade, which triggers whether or not the payoff is received. Barriers are typically less valuable than European or American options as the potential payoff can be restricted by the barrier. Some knock-out options pay a specified amount, often called a rebate, if the barrier is reached.
Bermudan	Option can be exercised at multiple predetermined dates prior to the maturity date of the contract.
Chooser	Gives the holder the right to select between payoff scenarios after the commencement of the contract, such as whether the option will be a call or put.

Cliquet (Ratchet)	Refers to a package of sequential options (or option strategies) sold as one product, eg eight three-month options, covering a two-year period. These options are of equal maturity and run consecutively to one another and the strike for each subsequent contract is set once the previous contract matures (each of these component options are known as a forward-start options). There may be additional features such as caps and floors on any payments, which are themselves options that prevent a payment from exceeding (cap) or falling below (floor) a predetermined level.
Compound	Grants the holder the right to enter into an option position at a future date eg, a “call on call” is an option to purchase a call option.
Digital (Binary)	The option payoff is either a fixed sum or zero.
European	The holder may exercise the option only on its date of maturity. This is the simplest type of option available.
Forward-start	The strike is not determined at the inception of the contract but rather at a future date. The strike level is usually dependent on the price of the underlying equity on the forward starting date. A combination of forward-start options is called a cliquet option.
Lookback	The option payoff depends on the minimum and/or maximum price of the underlying equity over the life of the option. As the potential payoff is always at least as high, and will usually exceed that of a similar European-style option, lookback options have higher premiums than similar European-style options.
Quanto	The payoff of an option is denominated in a different currency to that of the underlying equity. There are three main varieties: Currency-translated option: The option payoff is paid in a foreign currency using the prevailing spot rate at maturity. Foreign-strike option: The strike price of an option is denominated in a different currency to that of the underlying equity, eg an option on a US Dollar traded equity with a strike denominated in Japanese Yen. Quanto-option: The option payoff is paid in a foreign currency using a pre-specified exchange rate, which is usually the spot rate at inception.
Range accrual	This is a variation on a barrier option, where the payoff depends on the period of the option contract for which an underlying asset has spent within a certain price range.

19. Options can also derive their value from several underlying equities. There are many possible option types and for illustrative purposes some common examples are listed below:

Basket	The option payoff depends on a combination of the price changes of many underlying equities, eg, an option on all equities in an index.
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Best-of or worst-of	The option payoff depends on the best or worst performing equity in a basket.
Dispersion	The option payoff is related to the relative change in the price of equities to one another. Two common types are outperformance and rainbow options.
Exchange	The holder of this option has the right to exchange one equity for another in predefined proportions.
Knock-out components	When the price of an equity meets a certain level it is no longer considered when calculating the payoff of an option, ie, that part of the option is “knocked-out”.
Mountain range	These options are constructed through differing combinations of basket and range options.

Structured Products

20. Structured products can generally be viewed as a bond coupled with an equity option, however they can include any number of additional features too. Common features include:
- coupon payments that are linked to the price of an equity,
 - a notional amount may be at risk,
 - the notional may be repaid before maturity due to predetermined events affecting the underlying equity or by amortisation over time.
21. Structured products appeal to investors as they can be a source of regular income whilst potentially allowing them to profit from exposure to an underlying equity. A common structured product is the Equity Linked Note (ELN). As a structured product an ELN can be characterised by the combination of a bond and an equity option, namely a zero coupon bond guaranteeing a minimum principal repayment and an equity option that provides an additional source of return based on the price of a specified equity. This paper does not deal with the valuation of structured products as it is proposed to address these in future guidance.

Corporate issues

22. Derivatives are generally issued by sell side institutions for insurance, hedging or leverage purposes; however some types of derivative contract can be issued by any entity. This type of derivative contract is known as a corporate issue and is typically used by entities as a source of capital or in order to remunerate employees. Examples include:

Employee option	These are call options that are given to employees as a part of their overall remuneration. These are often forward-start options and typically the underlying equity will be that of the corporation that issued the derivative.
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Warrant	These are similar to call options, however unlike options, the derivative issuer is usually the corporation specified as the underlying equity of the contract. Interestingly this means that if a warrant is exercised the corporation can choose to issue new equity rather than having to buy existing equities in the market.
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Volatility derivatives

23. The management of volatility, ie the variability of equity prices over time, is a key reason for the existence of equity derivative markets. The underlying asset of this type of derivative is a measure of the volatility of an equity. One of the most common volatility contracts is known as a variance swap where one leg makes a fixed payment and the other pays an amount equal to the annualised realised daily variance of an underlying equity over the period. Another related swap is the volatility swap which uses the standard deviation of price changes in an equity, ie the square root of the variance, as its underlying.

Strategies

24. The term “strategy” is used in this paper to refer to a combination of derivatives and other assets designed to achieve a specific investment objective. As the value of a strategy is the sum of its parts, it is not considered to be a separate type of derivative contract and their valuation is not discussed in this paper.
25. There are many different ways by which derivatives can be combined to yield a strategy. A few commonly used examples are presented below in order to illustrate how derivatives can be combined with one another. However, given the extensive number of possible combinations of derivatives this is not intended to be an exhaustive list.

Butterfly	Combination of long and short call options. This strategy results in a small gain if the underlying equity price remains stable and generates a limited loss if the price fluctuates up or down. It is equivalent to the combination of a long straddle and a short strangle or vice versa.
Collar	Combination of a long equity position, a long put and short call. This strategy constraints gains and losses to a predetermined range.
Covered Call	Combination of a long equity position and a short call. This strategy replicates a short put position and increases the gain by the option premium below the call option strike and limits profit thereafter.
Protective Put	Combination of a long equity position and a long put. This limits losses whilst allowing for unlimited, but reduced gains.
Straddle and Strangle	Combination of a long call and a long put. Generates a loss if the price of the underlying equity remains stable and earns a gain if it moves up or down enough.

Valuation of forwards and futures

26. The valuation of a forward or future involves two conceptual steps:
- a) an estimate of the future price of the underlying equity; and
 - b) a calculation of the present value of the difference between the expected future price and the predetermined contract price of the equity.
27. In order to forecast the future price of an equity a number of assumptions are needed about the future and of these the “rational pricing” assumption is essential. Under this assumption similar assets will be priced similarly, ie two assets that pay the same cash flows will have the same price. It also means that, given the current price of an asset, the forward price can be explained in terms of what it is known as the “cost of carry”, which is the difference between the cost of owning an asset and the financial benefit of owning it. In general terms, cost of carry for a dividend paying equity is calculated as the interest rate paid to finance the purchase minus the income earned from income flows such as dividends and equity lending fees. For a non-dividend paying equity with no spread available from equity lending, the cost of carry is simplified to the interest rate paid to finance the asset.
28. The value of a forward or future contract, which is equal and opposite for each party, can now be calculated as the present value of the difference between the expected price of the underlying at maturity and the delivery price. For example, say a party has a long forward position to buy an equity at a future date at a predetermined delivery price. At a future point in time if the forward price of the asset is higher or lower than the delivery price, the contract will have a positive or negative value respectively to the party that holds the long position and an equal and opposite value to their counterparty.

Valuation of swaps

29. The value of a swap is the net present value of its expected cash flows. Rational pricing is assumed to hold, which means that in order for both legs of the swap to be arbitrage free, the terms of a swap contract are adapted at inception so that the net present value of the expected cash flows is equal to zero. However, after inception the value of the contract is unlikely to remain at zero as the underlying elements of the swap change.
30. Each leg of an equity swap is typically present valued independently and then netted against the other, except when the legs are interrelated, eg the financing leg is autocallable based upon the performance of the equity leg. The financing leg, which is usually a floating rate, is valued similarly to a floating rate bond, whereby expected coupon payments are forecasted using market implied forward and then discounted to a present value. The equity leg is priced using the techniques explained in the discussion on option pricing models below. The choice of model will depend on the structure of the swap and careful judgement is required when determining which valuation model to use.

Valuation of options

31. Three of the most widely used categories of equity option valuation models are discussed within this paper, namely:
 - a) The Black Scholes model and its extensions;
 - b) Alternative-diffusion models; and
 - c) Jump-diffusion models.
32. It is possible to value most equity derivatives with the three categories of model mentioned above. There are many considerations which can influence the choice of model, such as market convention, the purpose of valuation, computational complexity, time, and the quality of valuation inputs available. There are no right or wrong models and the decision on which to use requires consideration of a number of issues before determining if it is appropriate to use one over another, eg, it may be more appropriate to use a less sophisticated model with robust inputs than a more sophisticated model with less robust inputs.
33. Whilst the choice of model will depend upon the specific circumstances of a valuation, there are certain assumptions that are always needed when valuing an equity derivative. The most important of which, is an assumption of risk neutrality. In a “risk neutral world” investors are only interested in the expected return of an investment, irrespective of the risk. By making this assumption it can be assumed that there is no additional value attributed to the risk of an investment and as such the expected return of all the securities is the cost of carry, ie in a risk neutral world it is assumed that values have been adjusted, so that the expected return on all investments are the same and therefore equal to the risk free rate.
34. Investors in the real world will consider the risk of a derivative carefully; however the assumption of risk neutrality is still a very useful one to make. This is because when one assumes that investors are risk averse two counteracting effects take place. Firstly the expected return of the asset increases because an additional premium is required by investors to compensate them for the risk of the investment. Secondly the rate used to discount the payoffs of the derivative also increases. These two effects offset one another and therefore the solutions under an assumption of risk neutrality are valid under any set of risk preferences.

Black Scholes model

35. The Black Scholes model involves the creation of a dynamically managed “replicating portfolio” that replicates the cash flows of an asset by combining other financial instruments together into a portfolio. The value of the option is then equivalent to the value of its replicating portfolio.
36. The original Black Scholes model enabled the valuation of European options based upon the following assumptions:
 - Changes in equity prices follow a Geometric Brownian Motion (GBM). This means that equity prices change at random, follow a normal distribution and are driven by a deterministic and stochastic component, namely the expected return of an equity and the volatility of an equity respectively.
 - Equity price volatility is assumed to be constant.
 - The interest rate is constant and equal for all maturities.
 - There are no dividend payments during the life of the derivative.

- There are no transaction costs or taxes.
 - There are no riskless arbitrage opportunities.
 - All securities are perfectly divisible.
 - The short selling of securities is permitted, which is used to hedge the derivative by buying or selling the underlying equity in fractional amounts.
37. Subsequently the original model was adapted by different authors to enable the valuation of other types of option. Merton (1973) extended the Black Scholes model to allow for a continuous dividend yield. Black's formula (1976) gave the price of European options when the underlying security is a forward or futures contract. Ingersoll (1976) allowed transaction costs and taxes.
38. Given the assumptions listed above, the Black Scholes model facilitates the valuation of an option using the Black Scholes partial differential equation. This formula was originally created to value European options, however this has since been expanded to value other types of option.
39. Each type of derivative that can be valued using the Black Scholes model will have a unique differential equation of value. This is called a closed-form solution, where the value of a derivative can be calculated through a series of formulae. This allows variations in a derivative's value over time to be determined by solving the equation with updated inputs, which is very quick and convenient. Unfortunately, the Black Scholes model cannot value all types of derivative and in this case a closed-form solution cannot be obtained. When this occurs, it is necessary to rely on calculation techniques that provide approximations to the solution of this equation. This is often the case with complex derivatives which require numerical approximations to estimate their value where a closed form pricing solution does not exist. Specific resolution methods for the Black Scholes model are discussed briefly below and the general principles of model resolution are discussed later in this paper.

Black Scholes Model Resolution

40. There are three commonly used approaches to Black Scholes model resolution each of which is considered in further detail in the model resolution section:
- Analytical solution;
 - Monte Carlo; and
 - Finite differences.
41. The Black Scholes partial differential formulae is most applicable to vanilla options, however many closed-form analytical pricing formulas have been developed for more exotic options.
42. Monte Carlo simulation is a numerical method that is considered in option pricing when no closed-form solution is available. A Monte Carlo simulation can be used to emulate a wide range of stochastic processes and it is particularly useful when the derivative payoff is path-dependent and when it depends on more than one underlying asset.
43. The finite difference method is used to find a numerical solution to the differential equation. As in Monte Carlo simulation, the method is valid to solve not only the Black Scholes partial differential equation but a wider range of differential equations.

Limitations of Black Scholes

44. The Black Scholes model is the most widely used method of valuing options, however the model is predicated on some assumptions that may be deemed unrealistic in many situations. It is therefore important to be aware of the limitations of the Black Scholes and how each assumption drives the value of an option. This is particularly important to understand as some assumptions have a greater impact on the value indicated by the model than others.
45. The limitations of the Black Scholes model lie primarily in the assumptions that must be made in order to use it. The following assumptions are considered to be inappropriate in some valuation scenarios:
- The GBM assumption implies that equities follow a random walk, which means that the price of the underlying equity is just as likely to go up or down in price at a given moment in time. There is strong evidence to suggest that this is not the case.
 - Equity prices are assumed to be log normally distributed and returns assumed to be normally distributed. In practice many have observed that returns follow a leptokurtic distribution whereby returns exhibit large outliers more regularly than would be expected than if they were normally distributed, these are known colloquially as “fat tails”.
 - Equity price volatility is assumed to be constant, however price behaviour during a market crash would suggest this is not the case. While volatility can be relatively constant for short periods, it is not constant in the long run, eg higher than average volatility is often observed after a large change in prices.
 - Interest rates are assumed to be risk-free, constant and known. This can be approximated, eg by using Treasury Bills, however, these rates will change over time.
46. The Black Scholes model has been modified extensively since its inception; depending on the circumstances of a valuation these extensions should be used wherever the original model would lead to an unrepresentative value.

Alternative-diffusion models

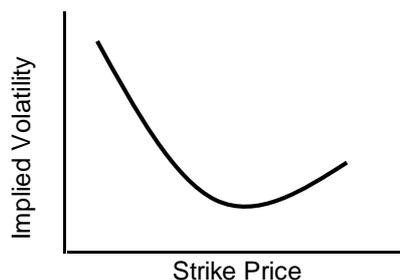
47. Another alternative is to assume that equity prices change continuously, as in the Black Scholes model, but to assume that this is driven by a process other than a GBM, ie an alternative type of diffusion.
48. One of the distinguishing characteristics of alternative-diffusion models relative to the Black Scholes model is in the treatment of volatility. As mentioned earlier, volatility is not constant in equity markets. Alternative-diffusion models utilise dynamic volatility modelling in an attempt to more closely model real market volatility. These can be classified in to three categories:
- time dependent volatility;
 - local volatility; and
 - stochastic volatility.

Time dependent volatility models

49. Implied volatility varies with an option’s expiration date as well as the strike price, consequently time dependent volatility models modify traditional constant volatility models to make volatility time dependent. Merton (1973) was the first to propose a formula for pricing options where volatility is a function of time.

Local volatility models

50. The chart below plots the implied volatility of a typical equity option against its strike price. This shows that implied volatility exhibits a convex curve, referred to as the “volatility smile”, which is caused by higher implied volatility for out-of-the-money options (the strike is far from the current price) relative to at-the-money options (the strike is close to the current stock price). This is because only a large price move in the underlying equity would result in a significant change in the value of an option that is far out-of-the-money. Local volatility models attempt to more closely mimic this relationship than time dependent models by expressing volatility as a function of both price and time.



51. There are several advantages to using local volatility models, that include:
- They can account for a greater degree of empirical observation and theoretical argument than time dependent and constant volatility models.
 - They can be calibrated to fit empirically observed data, which enables the consistent pricing of derivatives.
 - No additional source of randomness is introduced into the model; hence a unique value for the option exists.

Stochastic volatility models

52. Local volatility models are an improvement on time dependent volatility models, however they assume that volatility is directly dependent on the equity price. This is not supported by actual prices which typically exhibit “volatility clustering”; whereby large changes are typically followed by other large changes and small changes are typically followed by other small changes. Stochastic volatility models address this by allowing volatility to be governed by its own stochastic process. There are many commonly used stochastic volatility models some of which include the Johnson & Shanno, Scott, Stein & Stein, Heston and SABR models.
53. Stochastic volatility models are able to generate similar return distributions to those observed in the market, eg it produces return distributions that are leptokurtic (peaked) with “fat tails” compared to normal distributions. Moreover the asymmetry of the distribution’s tails can be controlled by the correlation between the spot stochastic process and the volatility process.
54. Stochastic volatility models have more than one source of randomness. Also, it is common for stochastic volatility models to not have closed form solutions for option prices, consequently option values are usually calculated by simulation, semi-analytical methods or numerical expansions. It may be appropriate to fix or restrict the iterations of one or more inputs to the model when calibrating the model in order to reduce complexity and limit the number of available combinations of inputs. Calibration is discussed in more detail later in the paper.

Jump-diffusion models

55. In a jump-diffusion model a “jump” term is superimposed on the GBM. The result is that prices are assumed to change continuously, as under the GBM of the Black Scholes model, however from time to time larger jumps may occur driven by the jump term. Unless volatility is extremely high, the probability of a large change in an equity price over a short time period is very small under a GBM. The addition of a “jump” term results in values that more closely resemble the “fat tails” that are observable in the market.
56. The “jump” term is generated by a Poisson process, which is a type of continuous stochastic probabilistic process. When using a jump-diffusion model an important consideration is the selection of appropriate jumps. The most appropriate choice will depend upon the circumstances and could include:
- downward jumps only, or include upward jumps as well;
 - many small jumps, or infrequent large jumps;
 - whether the jumps are of a deterministic (non-stochastic) size or if they should come from a distribution; or
 - whether a combination of jumps are appropriate.
57. As mentioned above, jump-diffusion models are appropriate when equity prices cannot be assumed to follow a smooth diffusion process, such as a GBM. In practice this means jump-diffusion models are particularly appropriate in the valuation of barrier options as well as other options with relatively short times to maturity.
58. It is important to note that the risk preferences of the investor have to be considered and as there are at least two sources of randomness in a jump-diffusion model there can be a range of potential values. There are several approaches to dealing with the additional complexity that this introduces in to the valuation process, however these are not considered in this paper.

The Greeks – Sensitivity Measures for Derivatives

59. As has been discussed above, derivative valuations are driven by a many underlying factors. The “Greeks” are a range of measures, generally annotated by letters from the Greek alphabet, which indicate the sensitivity of a derivative’s value to a change in a particular valuation input.
60. Measures of sensitivity are very useful indicators as that they not only signal whether the value of a derivative will increase or fall, but they also indicate the likely degree of change in the value of a derivative following a predetermined and small change in a valuation input.
61. The following is a short list of the most commonly used Greeks. For illustrative purposes the sensitivities discussed are in the context of options but the principles can be applied to all types of derivative.

Delta	Sensitivity of a derivative’s value with respect to a change in the price of the underlying equity. This is also known as the hedge ratio and can also be interpreted as the amount of underlying equity required to hedge a derivative position.
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Gamma	Sensitivity of the delta with respect to a change in the price of the underlying equity. This is a measure of how often a hedge will need to be rebalanced to maintain a delta-neutral position.
Rho	Sensitivity of a derivative's value with respect to a change in the interest rate.
Phi	Sensitivity of a derivative's value with respect to a change in the holding cost (dividend yield plus stock lending fee)
Theta	Sensitivity of a derivative's value with respect to the passage of time.
Vega	Sensitivity of a derivative's value with respect to volatility of the underlying asset.

62. For simplicity the discussion above has been in the context of individual derivative products, however sensitivities are also calculated on a portfolio basis. When an entity is party to more than one derivative contract it is important to consider their sensitivity as a portfolio. This is because the value of each derivative is likely to respond differently to a change in any common valuation inputs, eg in certain circumstances a change in a common valuation input may cause the value of two derivatives to move in equal and opposite values that effectively cancel one another out. The calculation of sensitivities at the portfolio level is particularly useful for those entities that manage large quantities of derivatives and enables them to better understand the true changes in value to which they are exposed and plan accordingly.

Model resolution

63. The discussion so far has focussed on the different types of models that can be utilised to value equity derivatives. All models will require resolution before they can be applied. There are many techniques available for model resolution and the choice will depend on the applicability of the method given the model used and the product being valued. The most commonly utilised methods of resolution are outlined below.

Analytical solution

64. Analytical solutions, also referred to as closed form solutions, are the preferred approach to model resolution wherever possible as they yield quick and accurate results. Unfortunately analytical solutions are only available for a few types of derivative contracts and models therefore this approach can only be used for a limited range of products.

Monte Carlo

65. The Monte Carlo can be particularly useful when there are multiple underlying equities or the price movement through time needs to be modelled. In the Monte Carlo uncertain valuation inputs are treated as random variables. Random numbers are used to simulate the changes in the underlying asset or assets, which in turn yields a sample value of the derivative given the relationship outlined in the model. This is repeated numerous times and an average is taken across all of the results, which is then discounted to give the current value of the derivative.
66. It is important to consider the random number generation process that will be utilised along with the Monte Carlo. This source is highly influential on the valuation and care must be exercised

to avoid programmes that could skew the outcome by not generating genuinely random number inputs.

67. The Monte Carlo can be computationally intensive and therefore time consuming. However the increase in computational time is approximately linear with the increase in variables (eg multiple assets). Therefore in terms of computational time, as more variables are analysed, the faster the Monte Carlo performs relative to other methods where the computational burden increases more quickly. Moreover, although the Monte Carlo is computationally intensive, this is due to the large number of simulations that have to be performed, rather than the complexity of the modelled expressions themselves. As a result it is relatively straightforward to make amendments to the model from a programming perspective.
68. As a numerical approximation, the Monte Carlo produces an error term; however this error can be measured and subsequently reduced by increasing the number of simulations. There are other techniques that may be used to reduce the error, but these are more difficult to apply across different derivative products. It can also be time consuming to compute the Greeks.

Trees

69. The future path of an underlying asset is modelled using a recombining tree, using discrete upward and downward steps over short time periods to approximate a continuous path. Probabilities are assigned to the up and down movements. The terminal price of the option is known, given the underlying asset price at maturity, and therefore it is possible to work back down the tree using the probabilities to obtain the current price of the contract. These methods are good for pricing early exercise features, but finite difference schemes tend to be superior in most respects.

Finite difference

70. Finite difference methods use a rectangular grid of specified points of asset price and time to model prices. This yields more stable valuations than a tree with the similar ability to price early-exercise contracts and products such as barriers. As with trees, the current value is inferred from the terminal value. These methods can be quick to apply, and extrapolation techniques can be employed to obtain more accurate prices. Evaluating the Greeks for hedging and risk management is similarly efficient. All of these benefits come at a price, which is limited dimensionality; practically speaking it is unfeasible above three dimensions (two asset processes plus time). Finite difference models are often used for vanilla American options but are less suitable for other exotic products such as path-dependent options.

Numerical integration

71. This covers two approaches that work in a similar manner, which are considered briefly below.
- Fourier/Laplace transform methods are relevant for certain stochastic volatility models as the pricing of vanilla instruments can be calculated through numerical integration, which is timely and accurate.
 - Quadrature methods directly evaluate derivative values by integrating probability distributions against payoffs. Much like trees and finite difference methods this is used primarily for low dimensionality problems, and for early exercise.

Model calibration and input selection

72. Once a model has been selected and resolved, inputs need to be selected and the model needs to be calibrated. Calibration is the process whereby inputs are selected in such a way so as to maximise the ability of a model to match market prices. Brief guidance for some of the most common inputs is offered below; this should also be read in conjunction with the commentary in IVS 250 *Financial Instruments*³.

Interest Rate

73. Interest rates are typically calculated through a process called bootstrapping. The process begins by collating the market prices of financial instruments of differing maturities, such as deposits rates, futures, forward rate agreements and interest rate swaps (IRS). These market prices are then converted into zero coupon discount factors, which are in turn transformed into spot rates and then plotted as a curve. This allows the calculation of appropriate interest rates over differing time periods.

Volatility

74. The volatility of an equity is a measure of the variability of its returns over time. The volatility of an equity cannot be directly observed and has to be calculated either through the use of a model or historical data analysis. Volatility that is calculated through the use of a model is referred to as implied volatility and it represents market expectations about the equity's future volatility. Implied volatility is generally the preferred option although it is not always possible to compute, in which case historical volatility data can be used instead.

Correlation

75. Correlation measures the strength of a linear relationship between one variable and another. There is a growing market in correlation and covariance trading, which is most developed for indices versus their constituents. The correlation can be implied by entering the prices of these products into models. For some products, a correlation term structure and/or a correlation skew may be evidenced. When market implied correlations are unavailable, historical correlations can be used instead.

Dividends

76. When estimating dividends, a distinction is usually made between the short and long term. In the short term, the entity's dividend policy, as well as the frequency and size of any dividends are taken into consideration. In the long term, it is common to calculate implied dividend yields from market prices, eg, implied dividend yields can be estimated using the put-call parity relationship for European options or by using dividend swap quotes. When implied dividend yields cannot be calculated, historical dividend yields can be considered instead. Additionally it is important to consider the proportion of any dividends that are paid in cash as this can have a significant impact on any subsequent valuation.

³ International Valuation Standards 2013, p.83 para C16-C19.

Holding Costs

77. If the equity lending fee is not observable in the market, it can be estimated by put-call parity from the prices of put and call options on the same underlying with the same strike and maturity.

Implementation

78. There are a vast selection of implementation approaches available, the choice of which depends on the type of entity, its resources and the end user.
79. Initial testing and model development may be performed using high-level languages and software⁴ as these have ready-to use libraries of numerical tools, and debugging is relatively straightforward.
80. For release code, a low-level programming language⁵ can be used to build a pricing library. These languages do not have much standard functionality, so they are harder to program, but are faster at the execution level. Scripting or higher-level languages⁶ use the underlying libraries for batch pricing and risk management on a portfolio level. Individual users who are not associated with the programming phase, will often access the library via a spreadsheet plug-in.
81. Additionally, there are a number of vendors offering pre-packaged libraries, often including a spreadsheet interface.

Option valuation models and their applicable products

82. This paper has referred to some commonly used valuation models and associated methods of model resolution. The following section will briefly list which models are generally used to value the types of option product discussed earlier in this paper. This list is neither comprehensive nor exhaustive.
83. The following table shows models which are commonly used to value different products. It is for information only and is not intended to prescribe that a listed model should always be used for a particular product. The choice of valuation model and method is a matter of professional judgment⁷. It is possible that another model or method that is not listed may be the most appropriate to use given the purpose and circumstance of the valuation being undertaken.

Black Scholes

Option Type	Method of model resolution
American	Trees, Finite Difference, Monte Carlo
Asian (Average)	Analytical solution, Monte Carlo
Barrier	Trees, Finite Difference, Monte Carlo

⁴ examples include: MATLAB® and Mathematica®

⁵ examples include: C/C++

⁶ examples include: Perl, Python or Java/C#

⁷ see IVS Framework, para 1

Bermudan	Trees, Finite Difference, Monte Carlo
Chooser	Analytical solution, Trees, Finite Difference
Cliquet (Ratchet)	Monte Carlo
Compound	Monte Carlo
Digital (Binary)	Analytical solution
European	Analytical solution, Trees, Finite Difference
Forward-start	Analytical solution, Monte Carlo
Lookback	Monte Carlo
Quanto	Analytical solution, Trees, Finite Difference
Range Accrual	Trees, Finite Difference, Monte Carlo

Alternative-Diffusion

Option Type	Method of model resolution
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American	Monte Carlo
Asian (Average)	Monte Carlo
Barrier	Monte Carlo
Bermudan	Monte Carlo
Cliquet (Ratchet)	Monte Carlo
Compound	Monte Carlo
European	Analytical solution
Forward-start	Monte Carlo
Lookback	Monte Carlo
Range Accrual	Monte Carlo

Jump-Diffusion

Option Type	Method of model resolution
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Digital (Binary)	Analytical solution
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Conclusion

84. This paper provides an overview of some of the most commonly used models and methods for equity derivative valuation. Judgement is always required in choosing the most appropriate model or method based on the facts and circumstances of the valuation and the over reliance on one model or method should be avoided. Particular care is required to ensure that the assumptions on which a model is based are as realistic as possible given the type of derivative, market conditions and the purpose for which the valuation is required. The following provisions of the *IVS Framework* are particularly relevant in this context:

- The exercise of judgement and objectivity,
- The expectation of competence,
- The need to consider the use of more than one valuation approach or method.